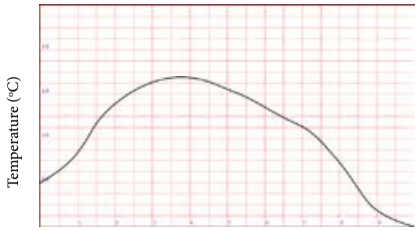
SECANT LINES AND TANGENT LINES

Use the graph below to estimate the rate at which the temperature of a liquid changes from:

Part A: 3:04 pm to 3:07 pm

Part B: 3:01 pm to 3:08 pm

At what rate is the temperature changing at 3:02 pm?



When working on this problem, students will read co-ordinates from a Cartesian grid; calculate average rate of change for the temperature using $\frac{\Delta T}{\Delta t}$; and consider average rate of change versus instantaneous rate of change.

However, more sophisticated approaches might discuss slope, secant lines, tangent lines and interval notation.

Some other things worth considering are:

- Once again the problem involves the concept of an instantaneous rate. However, in this case there is no exact answer because we only have the graphical form of the temperature function. Reading information from the graph introduces error.
- For drawing reasonably accurate tangents lines, it is often helpful to ask students to imagine the secant line which contains two points on the curve very close to the point of tangency

It is useful to ask students:

- Is the rate of change 'fairly constant' over the interval in Part A? Part B?
- For what period(s) of time is the liquid cooling?
- When is the liquid cooling the most quickly?

Here are two other problems students could tackle:

- How do you describe the steepness of a curve?

Possible solutions could involve a sequence of secant or tangent slopes summarized in a table, or perhaps transformed into a step function, or perhaps an equation resulting from a best fit curve program.

- How can you approximate the volume of a sphere using cylindrical shells?

Students need to know the volume of a cylinder. The Pythagorean Theorem is also required unless students read co-ordinates from graphs of circles being analyzed.

In summary, by viewing typical calculus questions through a 'high school lens', teachers can create opportunities that remove some of the mystery about calculus; help engage students through interesting problems; improve students use of function notation as a means of communication; reinforce many skills of the 8–12 curriculum and add value to their study; and help prepare students for calculus.

It would be a reasonable task for math departments to incorporate and find questions similar to the examples in this article. Based upon mathematical content and competencies, teachers could plan a variety of calculus problems for students to attempt as they transition through grade 8 to 12. For those that do take on this challenge, I hope you have time to share your plans and observations. There will be difficulties along the way, but this is a worthwhile endeavour that will be much easier if we learn and share together.

V

IMPLEMENTATION

Reggio-Inspired Math: A cross-district professional inquiry project

COMPILED BY JANICE NOVAKOWSKI ON BEHALF OF PARTICIPATING TEACHERS FROM RICHMOND, DELTA, SURREY, BURNABY, VANCOUER AND WEST VANCOUVER

Many primary teachers are enacting Reggio-inspired practices in their classrooms and have questions how these practices might enhance mathematics teaching and learning. Four Lower Mainland school districts collaborated to apply for a BCAMT grant to support professional inquiry in this area, with teachers from other districts joining in on the project.

Questions that guide our inquiry have included:

- How might Reggio-inspired practices be used in the area of mathematics?
- What does mathematical inquiry look and feel like in primary classrooms?
- How can we "make learning visible" for our youngest students?

Structures that we have used to support collaboration and professional learning across six districts include: two dinner meetings hosted in teachers' classrooms in Delta and Richmond; sharing of research-based and professional articles and resources; a group email list; a Google Doc with guiding questions and a place for sharing ideas and resources; sharing of professional learning and students' experiences on twitter using the hashtag #BCAMTreggio; some classroom visitation across districts; and making our professional learning visible through Twitter, blog posts and professional presentations across the Lower Mainland

At our second dinner meeting on May 26, an overview of Reggio-inspired practices was discussed and teachers from the Delta, Surrey and West Vancouver school districts shared their experiences. Janice Novakowski and Sandra Ball shared some mathematical content support materials they had compiled, based on emerging needs expressed by teachers involved in the project. Teachers realized that as they sat alongside students engaged with materials, they needed to deepen their own mathematical understanding of pertinent concepts such as counting, subitizing and decomposing so that they could both assess students' understanding as well as prompt them in their inquiries. The group involved in the project has decided to create some sort of online archive of the materials we have created and we look forward to sharing this information. Please contact Janice Novakowski for more information or follow the hashtag #BCAMTreggio on Twitter.

Following our sndergarten at Blair Elementary, has been examining the affordances of different materials to inspire inquiry and mathematical thinking. A documentation panel sharing her class' experiences with the question, "What can you discover about numbers?" is included at the end of this article.

Richmond School District

Michelle Hikida, a grades 2&3 classroom teacher at Diefenbaker Elementary began a mathematical inquiry with her students by asking the question, "What is a fraction?" She provided various math materials and loose parts to the students. Michelle noticed that students were able to use the materials to create the symbolic notation of fractions but weren't actually able to show their understanding or thinking about fractions initially. Previous math concepts she had used this same structure with had been familiar

to the students such as patterns or addition but fractions was a new concept in the formal curriculum at grade 3 and students did not have much experience with fractions at school. Michelle realized the students needed to build their understanding of fractions through the materials so she developed a series of provocations and inquiry questions based on the students' questions, misconceptions and current level of conceptual understanding.

This reflective, responsive stance to students and their thinking is one of the key practices we need to continue to highlight as we move forward with this project.

Surrey School District

After our first dinner meeting, Sandra Ball facilitated the creation of materials kits like the ones used in Richmond and Surrey teachers were able to "pilot" them for the rest of the year, thinking about how to develop provocations with the materials and finding ways to created opportunities to inspire mathematical inquiry. Teams of Surrey teachers also visited some Richmond classrooms in June and had an opportunity to debrief what they noticed with the classroom teachers.

Surrey School teacher reflections on our inquiry questions:

Kerri Hutchinson

I believe Reggio-inspired practices can be used in the area of mathematics because the philosophy sees the child as capable and competent of constructing their own learning. It also encompasses a social constructivist view of learning where children make sense of their world from learning from the people, places, and materials around them. This allows for open explorations, facilitated conversations, scaffolded learning and an entry point for each student. I believe Reggio inspired practices also allow students to reflect and return to their thinking during the documentation process. This process allows students to continually seek clarification and meaning from others and push their thinking forward. Lastly, I believe Reggio-inspired mathematics engages students in authentic math that connects them to their interests, their world, and their wonderings.

As I am very familiar with the foundational beliefs from Reggio Emilia I have been teaching this way in many other areas during 'project time'. During this BCAMT inquiry however, I learned how to connect math with the students' interests as well (through an 'actual size' project). I also learned that my students' thinking wasn't as focused because I had too many provocations occurring at once that addressed too many questions. I learned that I need to start with a big idea (as I would in Science and Socials inquiry) and then design provocations that explore thinking in this area.

Niki Leech

Sometimes I set up provocations and observe to see what the students do with the provocations and sometimes I set up provocations with inquiry questions based on one big idea. The questions are sometimes generated from students' wonders. For example, right now we are investigating number. I asked the question "Where do we find numbers?" which inspired students to go through the school looking for numbers and recording where they found numbers. We then shared their learning in a whole class discussion using the pictures that they took displayed on the Smartboard as well as including student voice. We then came up with student wonders sparked from the original question. The wonders included: What is the biggest number? Are there numbers smaller than zero? Who discovered numbers?

I set up 4 provocations to inspire students to investigate these inquiry questions.

1. *wooden numbers, dice, rulers, gems, felt pieces, paper, and the question "What do you know about numbers?"*
2. *100 charts, gems, paper and the question "What do you notice on the 100 chart?"*
3. *wooden numbers, felt pieces, frames, paper and the question "What is a big number?"*
4. *clip boards, ipads and the question "Where do you find numbers?"*

All of the students were engaged and excited. The classroom was full of talking. I circulated and asked questions, took pictures and documented what the students said.

I make learning visible in some of the following ways.

- *Take pictures of my students' discoveries and showing them on the Smartboard while they explain to their classmates about their learning. We reflect at the end of the math block and at the beginning of the next math block.*
- *Ask the students questions about what they are doing and document it. I sometimes put the documentation in a learning story or on a documentation panel.*
- *Encourage my students to put their learning on their own individual blogs.*
- *Encourage my students to teach each other.*
- *Use the class blog, twitter, and weekly newsletters so that parents can see what we are doing in the classroom.*

Delta School District

Alana Tesan and Kelli Lundie of Annieville Elementary shared their journey with transforming their classroom and embracing Reggio-inspired practices. This project provided an opportunity for them to extend what they were already doing in literacy to the area of mathematics. In particular, Alana and Kelli looked at the role of loose parts in mathematics. Through their inquiry they noticed the variety of entry points for students, high levels of engagement an communication and how students worked together as a community of learners when using the materials.

Burnaby School District

Teachers from the Burnaby School District had an afterschool session in June to engage in professional learning and consider ways that they may extend this project in their district. Supported by Angela Meredith and Ron Coleborn in their district, they aim to look at everyday math experiences that nurture mathematical inquiry. Burnaby will be hosting a half-day summer session on September 1.

West Vancouver School District

Misty Paterson of Irwin Park Elementary shared how she created an interdisciplinary study of family, involving mathematical inquiry. Misty and her kindergarten students also investigated balance in their world through a mathematical lens. At the end of this article are two documentation panels Misty created sharing her class' experience.

Vancouver School District

Gina Wong, the Early Learning and Literacy Coordinator for Vancouver, attended our second dinner meeting and included a session on Reggio-Inspired Mathematics at their district's Primary Piazza in May which generated much interest in this project. Gina will be hosting a full day summer institute examining Reggio-Inspired Mathematics in Vancouver on August 31 and we look forward to having Vancouver teachers involved in our collaborative inquiry for the next school year.

Significant outcomes that teachers have noted are that students are highly engaged in mathematics for long periods of time, are developing strong abilities to communicate their mathematical thinking and have an awareness that mathematics is diverse and varied in content. In terms of professional learning, teachers have noted a shift in their own pedagogy and a focus on listening to students and watching what they are doing. One teacher commented that her involvement in this project has been transformational in how she has thought about teaching and learning mathematics.

This group of teachers has been inspiring to work and learn with and the enthusiasm for this project has been growing. We will be bringing in more teachers and districts to this project for the next school year and are scheduling more frequent meetings so that we can share, collaborate and learn together.

The district teams involved in this project would like to thank the BCAMT for the grant that helped to establish this professional collaborative inquiry opportunity.

Exploring Numbers

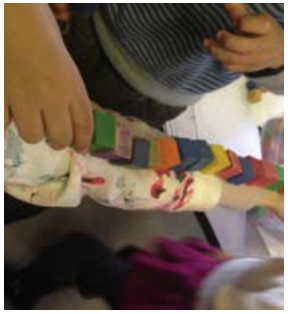
April 21, 2015

Our classroom has been participating in open-exploration learning. They are provided with a variety of materials at their desks as well as a provocation to help guide their thinking. This morning, we began our morning meeting by talking about our favourite numbers and where we can find them in the world. As they wandered around the classroom to choose which materials they would like to learn with, they noticed that the provocations are all the same—"What can you discover about numbers?" This is an open-ended question which allows ALL students to successfully begin their morning!

"What can you discover about numbers?"
Materials: light and dark coloured beads, dice, working mats
Student Reflection: "I roll the dice and have to count out the beads on my mat. I wanted to do more, so I made the beads in a spiral. Then I changed to also make a pattern."
Teacher Reflection: This student independently scaffolded their own learning! First, they stared with one-to-one correspondence (identifying the number on the dice and then counting out the beads). Then, they created a unique art display. Next, they added another mathematical element into their learning – patterns! Was this because they had enough time to explore with the materials? Would the student have continued to challenge their math thinking if it was a shorter amount of time?

"What can you discover about numbers?"
Materials: wooden numbers, dice, ten frames, counting beads, working mats
Student Reflection: "I heard someone else doing skip counting at the other table, so I thought I would try. It was easy at first, but then I ran out of 2's so we used upside down 5's. Then we also ran out of 4's, so we used 1's to cross over and then also used 7's to cross over."
Teacher Reflection: This student was able to open their minds and use the materials in a unique way. Taking inspiration from their classmates, they used creative thinking, problem solving skills and demonstrated a strong conceptual understanding of numbers.

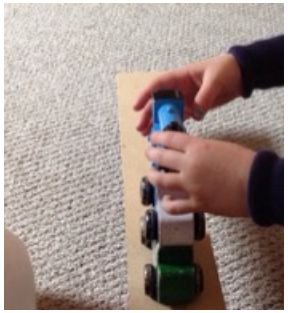
Using materials to tell "How Big?" stories…



Kindergarten, Irwin Park Elementary
West Vancouver

We have been exploring "family" and "interconnection" through a cross-curricular inquiry. I wondered how Reggio-inspired provocations in mathematics, specifically through measurement and geometry, might inform and depend our understanding. I noticed that the children naturally linked scientific and mathematical thinking and representation, and that these enriched our inquiry beautifully. The children remained curious about particular topics, such as "Who is taller?" revisiting this question with changing materials. It has "worked" to keep provocations "fresh", in response to the children's wonderings, but also to exercise flexibility and creativity when using familiar materials over the year (e.g., glass gems).

Where do we feel balance in our world?



Kindergarten, Irwin Park Elementary West Vancouver

"If you cut down trees, that's actually bad, 'cause we can't survive without air." -J, age 6

"You get paper from trees. If we cut down trees, we won't have paper." -A, age 5

The question of balance within our connection to the natural world emerged through a discussion about trees. I wondered how mathematical provocations involving the inquiry, "What does balance look like, sound like, and feel like?" might inform the children's thinking. One of my favourite responses was, "Balancing feels like slow motion. I have to do it carefully." I then asked the children, "What might it mean to go in slow motion when we make decisions to cut trees or take things out of nature?" When we see the power of "slow learning", I wonder how these questions and connections can continue to live on after this year as children leave our classroom and enter new grades and groupings.

IMPLEMENTATION

Keep it Simple: How to begin your guided math problem

BY KIM CLINE

So you've decided to try a guided math approach to teaching. Fantastic! Perhaps you've read some literature, attended a workshop or inservice, or typed the keyword into a Pinterest search. No doubt you've seen the overabundance of information at your fingertips including flashy themed commercially prepared group cards, stations, games and activities, student response forms, and teacher records. Maybe you've perused the endless resources and packages available through teacher created websites. The question remains, how to start?

Simply put, Guided Math, like its literacy counter part, involves an independent task for accountability, small group guided instruction by you the teacher, and a limited number of activities to be completed as a pair, triad, or individually. It requires a great deal of modeling, followed by a gradual release of responsibility, and above all, clear expectations for behaviour.

In the beginning, your time is spent introducing simple games and activities to the whole class, modeling the expectations, and practicing what it looks like and sounds like during math stations (what I call it in my grade 4 class). A behaviour anchor chart is useful for this process. Your role is to circulate and correct behaviours, provide feedback and encouragement, and to get a sense for the types of activities your students enjoy. Pulling a small group at this time isn't recommended.

Once you've established a routine and feel confident that the students understand what's expected of them during guided math, you can start to pull your groups. At this stage it might look like, one rotation of small group, math game with a partner, and math with technology (iPad or web-based game). Once this routine has been established, you can try to fit in a second rotation, and perhaps an additional station or task. Ideas include math journals, individual practice, open-ended questions, or task cards.

When forming groups, it's important to have a clear objective of what you'd like to accomplish. Perhaps you need to spend time building number sense with struggling math students. Maybe you have a group that's ready for a challenge, or there is a concept that you do not wish to teach as a whole group. The possibilities are endless. What's important is that you make the most of this fantastic teaching opportunity. It's intense, focused, and extremely rewarding. So much so, that you may lose touch with what the other students are doing. This is ok, you've practiced and the students are clear on what the expectations are. Let them go!

You'll need some basic supplies with you at your group table. A small supply of manipulatives, base 10 blocks, ten frames, a variety of dice (place value, decahedron) and individual white boards or blank paper in sheet protectors work well. You'll need some form of record to make notes on what the students are able to do. This doesn't have to be fancy. Sticky notes, lined paper, graphic organizer, or blank class lists are all effective. It's about finding what works for you. I personally like sticky notes.

Who's doing what? Forming groups for math is an ongoing, fluid, and evolving process. Consider math abilities of course, but also consider social-emotional factors. You need to create an environment that will allow you to work with your small group. It may require several attempts to coordinate kids in ways to keep them focused. Some students may not be able to handle working in pairs; they may need to use technology or individual activities more frequently. How you post the groups is a personal choice and there are plenty of examples out there.

Individual accountability is an important requirement of any math program. You need to keep a formative assessment of what students are able to do independently. A quick, mental math activity, sample of questions, or opening paper-pencil task is an important component in the guided math approach. This task can be done before starting rotations, during a rotation, or after as a mini-lesson. The purpose is to track student progress, which will eventually determine how your groups may be formed or the frequency of how often you see a group.

Math stations, or centres at the primary level, are the heart and soul of the guided math approach. There are several important things to consider such as storage of materials, use of classroom space, and the types of activities, variety, and quality of the games.

You should consider storing supplies in a central location where students can easily access games and any items needed to play them. Containers filled with cards, dice, counters, bingo chips, etc. can be easily obtained from the dollar store. In my class the top bin contains "active" games that match the current math concepts. They are cycled and refreshed on a regular basis. There are some basic computational games that remain available most of the year. Games not in use are stored in a filing crate according to concept. Students are able to play games on the floor around the classroom, or at their table groups or desks. To reduce noise, felt squares can be used to soften the dice, and socks are used to wipe off the dry erase marker. Students are encouraged to spread out to reduce potential behaviour issues and reduce noise.

Games, games, and more games! There are so many to choose from and it can be overwhelming. Simple is best. Kids don't need colour-graphics, laminated, flashy commercial items. They need simple, straight–forward, and quick to set up games. If you're just starting